ABSTRACT

Dissertation for the degree of Doctor of Philosophy (PhD)
6D060400 – PHYSICS

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Generalized Models of Gravity Theory and Black Holes

Relevance of the topic. Modern cosmology and gravity physics are characterized by fundamental challenges aimed at explaining the large-scale evolution of the Universe, the physical nature of its accelerated expansion, and the extreme properties of black holes. High-precision observational data obtained over recent decades, including the cosmic microwave background (CMB – Cosmic Microwave Background), Type Ia supernovae (SN Ia – Type Ia supernovae), baryon acoustic oscillations (BAO – Baryon Acoustic Oscillations), and gravitational lensing results, despite confirming numerous successes of the standard cosmological model ΛCDM, have revealed a number of unresolved issues. One of the most significant among them is the discrepancy in Hubble constant values obtained at high and low redshifts, known as the Hubble tension.

To address this problem, modified theories of gravity and alternative cosmological models are actively being explored. One such approach is quasi-steady state cosmology (QSSC – Quasi Steady State Cosmology), based on the oscillatory nature of the Universe's scale factor. This model is distinguished by the absence of an initial singularity and its ability to phenomenologically describe the Universe's dynamics at late stages of its evolution. Within the QSSC framework, the oscillatory temporal evolution of the Hubble parameter can lead to an increase in its value at low redshifts, which is considered a potential mechanism for alleviating the Hubble tension.

Alongside cosmological aspects, the investigation of geometric, optical, and thermodynamic properties of black holes serves as one of the key tools for testing gravity theories. Direct observations of shadows of supermassive black holes M87* and Sgr A* under the Event Horizon Telescope project have opened new opportunities for comparing theoretical models with astrophysical data. In this context, the analysis of black holes in spacetimes with extra dimensions, as well as non-singular black hole models, represents a particularly relevant direction in contemporary research.

Aim of the dissertation research. The dissertation research is directed at exploring the potential for phenomenological alleviation of the Hubble constant tension based on the quasi-steady state cosmological model QSSC, as well as conducting a comprehensive analysis of the shadow characteristics of five-dimensional Reissner-Nordstrem-anti-de Sitter black holes (RN-AdS$_5$) and the geometric and thermodynamic properties of non-singular Bardeen and Bardeen-Yang-Mills black holes within Einstein-Gauss-Bonnet (EGB) gravity.

To achieve the stated aim, the following tasks were addressed during the work:

1. Analysis of the oscillatory nature of the Universe's evolution within the QSSC model and evaluation of the potential for phenomenological alleviation of the Hubble parameter tension between values obtained at high redshifts (CMB/Planck) and low redshifts based on observational data (OHD – Observational Hubble Data, SN Ia, BAO).
2. Calculation of the RN-AdS$_5$ black hole shadow in plasma and vacuum environments, determination of the influence of charge, cosmological constant, and plasma parameters on the shadow radius, as well as qualitative comparison of the obtained shadow characteristics with observational data for supermassive black holes M87* and Sgr A*.
3. Investigation of the thermodynamic properties and critical phenomena of the stationary Bardeen-type black hole solution within EGB gravity.
4. Derivation of the stationary Bardeen-Yang-Mills black hole solution and analysis of its horizon structure, thermodynamic properties, as well as deviations from the Bekenstein-Hawking area law.
5. Statistical estimation of QSSC model parameters through comparison with contemporary observational data using the Markov Chain Monte Carlo method.

Objects of research include the quasi-steady state cosmological model QSSC and the oscillatory nature of the Hubble parameter; the RN-AdS$_5$ black hole shadow and null geodesic dynamics; thermodynamic and phase properties of the Bardeen black hole within EGB gravity; geometric and thermodynamic characteristics of the Bardeen-Yang-Mills black hole.

Research methods. Analytical methods (Friedmann-Lemaître-Robertson-Walker metric, Einstein and EGB equations, null geodesic equations), numerical methods (statistical analysis based on MCMC), geometric-optical methods (black hole shadow modeling), thermodynamic analysis.

Scientific novelty encompasses the following main results:

1. The capability of the QSSC cosmological model to alleviate the Hubble tension is demonstrated through quantitative analysis of the oscillatory nature of the scale factor and comprehensive investigation using MCMC. Based on observational data from OHD, Pantheon (SN Ia), and BAO, calibration of the model parameters (α, β, η, h) was conducted, resulting in consistency with observations at low redshifts at the 1σ level. The obtained results indicate that the QSSC model naturally reproduces a phantom-like state of effective dark energy $\omega_{DE}(0) < -1$ without introducing additional fields and asymptotically aligns with ΛCDM cosmology at high redshifts.
2. The RN-AdS$_5$ black hole shadow is thoroughly analyzed in plasma and plasma-free environments, including a complete parametric study of the influence of charge q, cosmological constant Λ, and plasma parameters on the shadow radius. The obtained results are qualitatively compared with observational data for supermassive black holes M87* and Sgr A*.

3. The thermodynamic properties of the Bardeen black hole within EGB gravity are analyzed in full parametric form; heat capacity, Gibbs energy, and critical phenomena in the extended P-V phase space are determined. The structure of phase transitions is examined in detail in the extended P-V phase space, where the black hole's thermodynamic behavior exhibits van der Waals characteristics typical for anti-de Sitter spaces.

4. Based on the stationary Bardeen-Yang-Mills black hole solution, a comprehensive analysis of the horizon structure, thermodynamic quantities (temperature, entropy, heat capacity), and radiation intensity is performed. The influence of nonlinear electrodynamics and the Yang-Mills field on the formation of non-singular black hole properties is established.

Structure and volume of the dissertation. The work consists of an introduction, four chapters, a conclusion, and a list of references. It contains 30 figures, 165 formulas, 23 tables, 203 references, and 98 pages.

Main provisions defended:

1. Oscillatory solution of the scale factor in the QSSC model.

It is shown that the oscillatory nature of the scale factor in the QSSC model leads to an increase in the Hubble parameter H(t) near the modern epoch, naturally forms a phantom-like state of effective dark energy $\omega_{DE}(0) < -1$, and aligns with H_0 values obtained at low redshifts at the 1σ level. The analysis based on observational data from OHD, Pantheon (SN Ia), and BAO using MCMC confirms that the oscillatory mechanism in the QSSC model phenomenologically alleviates the Hubble parameter tension and asymptotically aligns with ΛCDM cosmology at high redshifts.

2. Shadow of the RN-AdS$_5$ black hole.

For the charged RN-AdS$_5$ black hole, the photon sphere radius and shadow radius are analytically derived. It is shown that in vacuum, the shadow radius decreases with increasing charge q and cosmological constant Λ, whereas in a plasma environment, it increases with increasing charge q and plasma parameter k. The obtained results qualitatively align with observational data on the shadows of supermassive black holes M87* and Sgr A*.

3. Bardeen black hole in EGB gravity.

Within EGB gravity, an exact solution for a non-singular Bardeen-type black hole associated with nonlinear electrodynamics is obtained. The boundedness of curvature invariants throughout spacetime indicates the absence of a singularity. In the extended phase space, thermodynamic characteristics analogous to the van der Waals system are identified: a first-order phase transition, critical point, and critical exponents corresponding to mean field theory

$\alpha = 0, \quad \beta = \frac{1}{2}, \quad \gamma = 1, \quad \delta = 3.$

4. Generalized Bardeen-Yang-Mills black hole solution.

A new static spherically symmetric Bardeen-Yang-Mills black hole solution is derived, integrating nonlinear electrodynamics and the Yang-Mills field. This solution reproduces the Bardeen (ν = 0), Yang-Mills (ε = 0), and Schwarzschild (ν = 0, ε = 0) metrics in limiting cases. The boundedness of curvature invariants

confirms the absence of a central singularity. Thermodynamic analysis establishes a violation of the Bekenstein-Hawking area law due to nonlinear field effects, as well as regions of stability and a phase transition associated with maximum temperature.

Author's personal contribution lies in direct involvement in all research stages: derivation of field equations, analytical analysis of geometric and thermodynamic properties, execution of numerical calculations, comparison of the QSSC model with data, and interpretation of results. All key analytical conclusions and MCMC calculations were performed personally by the author.

Approbation of the work. The results were presented at conferences: «Gylym jane bilim-2019» (Astana, 2019), II Conference of the Kazakh Physical Society (Almaty, 2019), VI International Conference «Role of Physical-Mathematical Sciences in Modern Educational Space» (Atyrau, 2021).

Published 10 scientific works, including 4 articles in **WoS/Scopus** journals:

- *Physics of the Dark Universe* (2023, IF 5.8, Q1);
- *International Journal of Modern Physics A* (2023, IF 1.3, Q3);
- *European Physical Journal Plus* (2023, IF 2.9, Q2);
- *Annals of Physics* (2023, IF 3.0, Q2);

as well as 1 article in a journal recommended by the Committee for Control in the Sphere of Education and Science of the Ministry of Education and Science of the Republic of Kazakhstan and 5 conference materials.

Connection with scientific research plans. The work was conducted within two budget projects of the Ministry of Education and Science of the Republic of Kazakhstan:

«Study of the Universe's Evolution Based on Generalized Theories of Gravity» (AR09261147, 2021-2023);

«Study of Cosmology in Metric-Affine Gravity Theory» (AR09058240, 2021-2023).

Theoretical and practical significance. The results obtained in the dissertation can be utilized to refine the geometric and thermodynamic characteristics of non-singular black hole models, as well as to evaluate the influence of physical parameters (field components, charge, mass, cosmological constant) on their properties. The calibration of QSSC model parameters based on modern observational data provides a phenomenological mechanism for interpreting the Hubble constant tension. Results pertaining to the photon sphere and shadow radius of RN-AdS$_5$ and Bardeen-type black holes can be applied in comparing theoretical models with astrophysical observations of supermassive black holes (M87*, Sgr A*), as well as in further studies in cosmology and gravity physics.